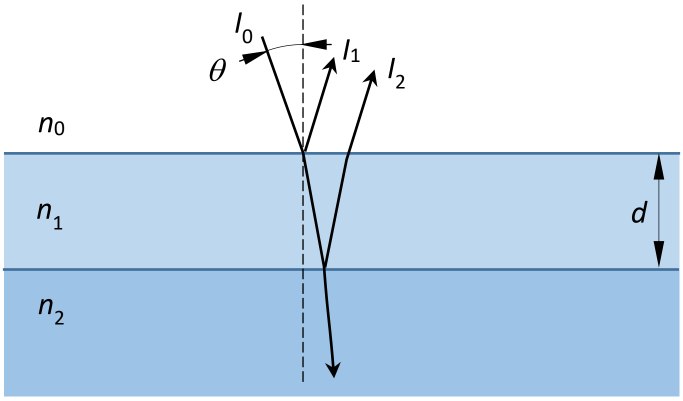
PRACTICE 10

Dr. Gábor Erdei, 26-11-2025

1 Analysis of an anti-reflection coating by using two-beam interferometry



$n_2 > n_1 > n_0$; λ_0 = 550 nm, in case of normal incidence ($\theta \approx 0$)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \tag{1}$$

$$OPD = n_1 \cdot 2d \tag{2}$$

$$\delta = \Phi_2 - \Phi_1 = \left(OPD\frac{2\pi}{\lambda_0} + \pi\right) - \pi = n_1 \cdot 2d \cdot \frac{2\pi}{\lambda_0} \tag{3}$$

In order to reduce reflection, the two waves have to meet in opposite phase:

$$\delta = \pi + 2\pi \cdot m \quad ; \quad m \in \mathbb{N} \tag{4}$$

If m = 0:

$$n_1 \cdot 2d \cdot \frac{2\pi}{\lambda_0} = \pi \quad \rightarrow \quad n_1 \cdot d = \frac{\lambda_0}{4}. \tag{5}$$

For a perfect suppression (destructive interference) the two waves should be of equal intensity. We assume the reflectance on both surfaces to be small, so the incident beam (I_0) and the beam reflecting on the second surface pass the first one with about no attenuation.

$$I_1 = I_0\left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2 \quad \text{and} \quad I_2 \approx I_0\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{6}$$

$$I_1 = I_2 \quad \Rightarrow \quad \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{7}$$

Since both reflection coefficients are of the same sign:

$$\frac{n_0 - n_1}{n_0 + n_1} = \frac{n_1 - n_2}{n_1 + n_2} \tag{8}$$

$$n_0 n_1 + n_0 n_2 - n_1 n_1 - n_1 n_2 = n_0 n_1 - n_0 n_2 + n_1 n_1 - n_1 n_2 \tag{9}$$

$$n_0 n_2 - n_1 n_1 = -n_0 n_2 + n_1 n_1 \tag{10}$$

$$2 n_1 n_1 = 2 n_0 n_2 \tag{11}$$

$$n_1 = \sqrt{n_0 n_2}\,. \tag{12}$$

If n_0 = 1.0 (air), and n_2 = 1.519 (Schott BK7 glass), then n_1 = 1.232. The refractive index of MgF_2 is 1.379. With this d = 99.7 nm (then $\delta = \pi$). The reflectance is:

$$R = \frac{I}{I_0} = \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2 + \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 + 2\frac{n_0 - n_1}{n_0 + n_1}\frac{n_1 - n_2}{n_1 + n_2}\cos\delta\,, \tag{13}$$

where $\cos\delta = -1$ now, due to destructive interference.

$$R = \left(\frac{1 - 1.379}{1 + 1.379}\right)^2 + \left(\frac{1.379 - 1.519}{1.379 + 1.519}\right)^2 - 2\frac{1 - 1.379}{1 + 1.379}\cdot\frac{1.379 - 1.519}{1.379 + 1.519} = 1.23\%. \tag{14}$$

That is, the approximately 4% reflectance of a BK7 glass measured at normal incidence has been reduced about to its one quarter.

2 Determining the period of a diffraction grating necessary for a given resolution

If the longitudinal modes of a laser diode are separated by $\Delta\lambda$ = 0.05 nm, the wavelength of one mode is λ = 635 nm, then what grating constant (a) of a diffraction grating should we choose, so that the grating can resolve the modes in its second order (m = 2)? Let the lateral size of the grating be D = 5.0 mm, the medium in which we are investigating is air (n = 1).

$$\delta = \frac{2\pi}{\lambda}\sin\theta_m \cdot a = m2\pi \tag{15}$$

m = 2:

$$\sin\theta_2 = \frac{2\lambda}{a} \tag{16}$$

$$I(\delta) = I_1\left(\frac{\sin(\delta N/2)}{\sin(\delta/2)}\right)^2 \tag{17}$$

The phase distance ($\Delta\delta$) of the first zero relative to the m^{th} maximum ($\delta = m2\pi$) is:

$$\frac{(\Delta\delta + m2\pi)N}{2} = \frac{(m2\pi)N}{2} + \pi \tag{18}$$

$$\Delta\delta N/2 = \pi \;\rightarrow\; \Delta\delta = \frac{2\pi}{N} \tag{19}$$

$$\Delta\delta = \frac{2\pi}{\lambda}\Delta\sin\theta_2 \cdot a = \frac{2\pi}{N} \tag{20}$$

$$\Delta\sin\theta_2 = \frac{\lambda}{a\cdot N} \tag{21}$$

How much does the direction of the m = 2 maximum change, if the wavelength is $\lambda + \Delta\lambda$?

$$\sin\theta_2 + \Delta\sin\theta_2 = \frac{2(\lambda + \Delta\lambda)}{a} \tag{22}$$

According to the Rayleigh criterion this equals the position of the first zero calculated above:

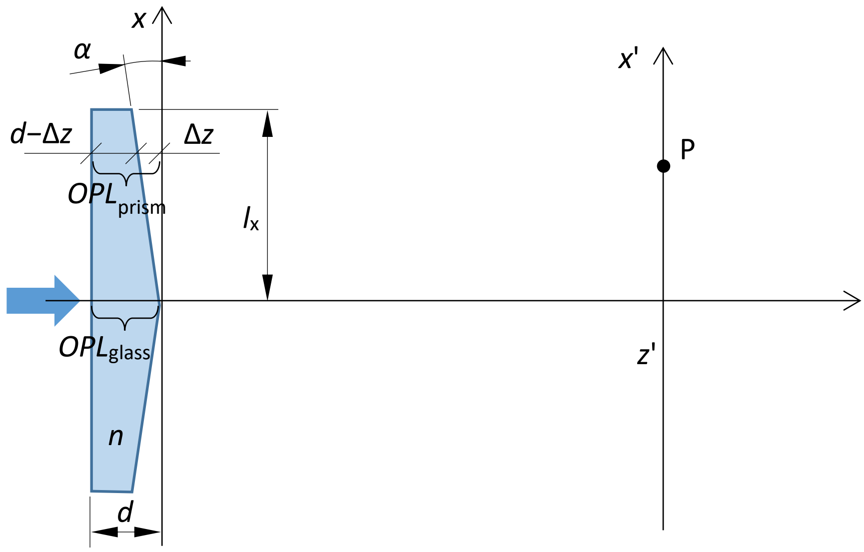
$$\frac{2\lambda}{a} + \frac{\lambda}{a\cdot N} = \frac{2(\lambda + \Delta\lambda)}{a} \tag{23}$$

$$2\lambda + \frac{\lambda}{N} = 2\lambda + 2\Delta\lambda \tag{24}$$

$$\frac{\lambda}{2\Delta\lambda} = N \;\rightarrow\; \frac{\lambda}{2\Delta\lambda} = \frac{D}{a} \;\rightarrow\; a = \frac{2D\Delta\lambda}{\lambda} = 0.787\ \mu\text{m}. \tag{25}$$

I.e. we need to use a grating of $1/a$ = 1270 lp/mm. The above value can be obtained again directly from the formula of the resolving power $\mathcal{R}$ (see lecture).

3 Determining the far-field diffraction of a Fresnel biprism



$$OPD = OPL_{\text{prism}} - OPL_{\text{glass}} \tag{26}$$

Hence, along the optical axis *OPD* = 0.

$$OPL_{\text{glass}} = nd \tag{27}$$

$$OPL_{\text{prism}} = (d - \Delta z)n + \Delta z \tag{28}$$

$$OPD = \Delta z - \Delta z \cdot n = \Delta z(1 - n) = x \tan\alpha \cdot (1 - n) \tag{29}$$

$$E(x) = e^{ik\cdot OPD} = e^{i\frac{2\pi}{\lambda_0}\cdot x\tan\alpha\cdot(1-n)} \quad , \qquad \text{if } x \geq 0 \tag{30}$$

$$E(x) = e^{ik\cdot OPD} = e^{-i\frac{2\pi}{\lambda_0}\cdot x\tan\alpha\cdot(1-n)} \quad , \qquad \text{if } x < 0 \tag{31}$$

$$b \triangleq -\tan\alpha \cdot (1 - n) \tag{32}$$

$$E(x') = \frac{e^{ikz'} \cdot e^{i\frac{kx'^2}{2z'}}}{\sqrt{i\lambda z'}} \int_{\Sigma} E(x) \cdot e^{-i\frac{kx'}{z'}x} dx \tag{33}$$

$$\int_{\Sigma} E(x) \cdot e^{-i\frac{kx'}{z'}x} dx = \int_{0}^{l_x} e^{-ikbx} \cdot e^{-i\frac{kx'}{z'}x} dx + \int_{-l_x}^{0} e^{ikbx} \cdot e^{-i\frac{kx'}{z'}x} dx \tag{34}$$

$$\int_{0}^{l_x} e^{-ikbx} \cdot e^{-i\frac{kx'}{z'}x} dx = \frac{1}{-ikb - i\frac{kx'}{z'}} \left[e^{-ikbx} \cdot e^{-i\frac{kx'}{z'}x} \right]_0^{l_x} = \frac{1}{-ikb - i\frac{kx'}{z'}} \left(e^{-ikbl_x} \cdot e^{-i\frac{kx'}{z'}l_x} - 1 \right) =$$

$$= \frac{1}{-ikb - i\frac{kx'}{z'}} e^{-ikb\frac{l_x}{2}} \cdot e^{-i\frac{kx'}{z'}\frac{l_x}{2}} \left(e^{-ikb\frac{l_x}{2}} \cdot e^{-i\frac{kx'}{z'}\frac{l_x}{2}} - e^{ikb\frac{l_x}{2}} \cdot e^{i\frac{kx'}{z'}\frac{l_x}{2}} \right) =$$

$$= \frac{2}{-kb - \frac{kx'}{z'}} e^{-ikb\frac{l_x}{2}} \cdot e^{-i\frac{kx'}{z'}\frac{l_x}{2}} \sin\left(-kb\frac{l_x}{2} - \frac{kx'}{z'}\frac{l_x}{2} \right) =$$

$$= \frac{l_x}{\frac{\pi b}{\lambda_0} l_x + \frac{\pi x'}{\lambda_0 z'} l_x} e^{-ikb\frac{l_x}{2}} \cdot e^{-i\frac{kx'}{z'}\frac{l_x}{2}} \sin\left(\frac{\pi b}{\lambda_0} l_x + \frac{\pi x'}{\lambda_0 z'} l_x \right) =$$

$$= l_x e^{-ikb\frac{l_x}{2}} \cdot e^{-i\frac{kx'}{z'}\frac{l_x}{2}} \operatorname{sinc}\left(\frac{b}{\lambda_0} l_x + \frac{x'}{\lambda_0 z'} l_x\right). \tag{35}$$

The minute constant phase term is present because at point $x = l_x/2$ the prism surface takes $z = l_x/2 \cdot \tan\alpha$ value instead of zero. The linear phase term is the wavefront of an oblique plane wave (observing at its intersection with the optical axis), which occurs because of the prism surface being shifted by $l_x/2$ from the axis. The shift of the sinc function corresponds to the obtained diffraction spot having an off-axis position:

$$\frac{x'}{\lambda_0 z'} l_x + \frac{b}{\lambda_0} l_x = 0 \;\; \rightarrow \;\; \frac{x'}{z'} = -\tan\alpha \cdot (n-1) \tag{36}$$

Which is in perfect correspondence with the law of refraction, when α is so small that

$$\sin\alpha \approx \tan\alpha \approx \alpha \;\; [\text{rad}]. \tag{37}$$

The wavefronts perpendicular to the direction determined by equation (36) are represented by the complex factor preceding the (33) Fraunhofer integral from which it was multiplied out in the form of a spherical wave centered on the aperture. When the distance of the diffraction spots taking place on the analysis screen is smaller than the Nyquist aperture, i.e. they significantly overlap, then interference occurs between them.