Physicist / Engineer physicist BSc, year III / semester 1
"Optics problem solving" – practice notes

# PRACTICE 3

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## 1. Reflectance, transmittance

$$R = |\rho|^2 \;\; ; \;\; T = |\tau|^2 \cdot \frac{n' \cos\theta'}{n \cos\theta} \tag{1}$$

## 2. Application of the Fresnel formulae: normal incidence from air onto glass

Refactive index of air $n$ = 1.0, one typical optical glass (BK7) $n'$ = 1.519 (at 550 nm wavelength). At normal incidence what is the surface reflectance?

$$R = \left(\frac{n - n'}{n + n'}\right)^2 = 0.206^2 = 4.2\% \tag{2}$$

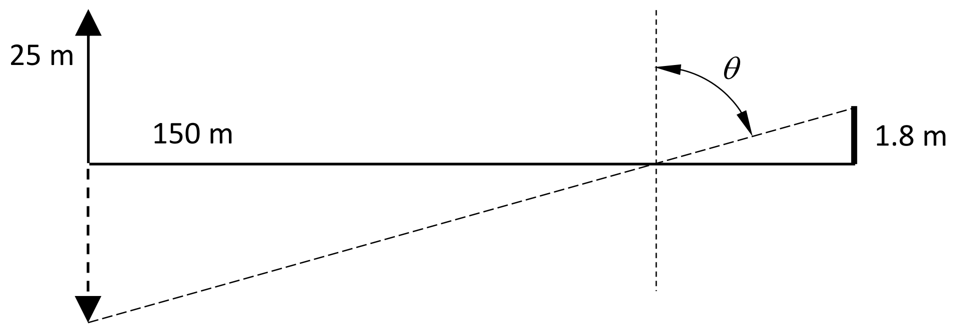
## 3. Application of the Fresnel formulae: normal incidence from air onto aluminum

Refactive index of air $n$ = 1.0, complex refractive index of aluminum $\tilde{n}'$ = 1.0 – $i$·6.6 (at 550 nm wavelength). At normal incidence what is the surface reflectance?

$$R = \left|\frac{n - n'}{n + n'}\right|^2 = \frac{1.0 - 1.0 + i6.6}{1.0 + 1.0 - i6.6} \cdot \frac{1.0 - 1.0 - i6.6}{1.0 + 1.0 + i6.6} = \frac{6.6^2}{2.0^2 + 6.6^2} = 91.6\% \tag{3}$$

## 4. Application of the Fresnel formulae: oblique incidence onto water surface

Standing at one bank of a 150 m wide lake we are observing the water reflection of the 25 m high tower of a castle situated on the other bank of the lake. Let us calculate the angle of incidence corresponding to the tip of the tower, and the reflectance for s- and p-polarization. Refractive index of water at 550 nm $n'$ = 1.334, that of air $n$ = 1.0, our visual height is 1.8 m.




$$\theta = \mathrm{atg}\left(\frac{150}{1.8 + 25}\right) = 79.9° \;\; \rightarrow \;\; \cos\theta = 0.176 \tag{4}$$

$$\cos\theta' = \sqrt{1 - \left(\frac{\sin\theta}{n'}\right)^2} = 0.675 \tag{5}$$

$$\rho_\mathrm{s} = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'} = \frac{0.176 - 1.334 \cdot 0.675}{0.176 + 1.334 \cdot 0.675} = -0.673 \tag{6}$$

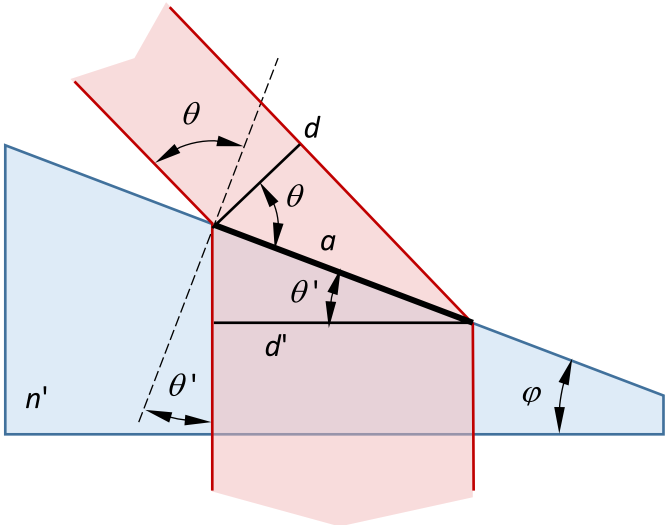
$$\rho_\mathrm{p} = \frac{n'\cos\theta - n\cos\theta'}{n'\cos\theta + n\cos\theta'} = \frac{1.334 \cdot 0.176 - 0.675}{1.334 \cdot 0.176 + 0.675} = -0.484 \tag{7}$$

$$R_\mathrm{s} = \rho_\mathrm{s}^2 = 45.3\% \tag{8}$$

$$R_\mathrm{p} = \rho_\mathrm{p}^2 = 23.4\% \tag{9}$$

## 5. Anamorphic prism: a change in beam size

We analyze one part of an anamorphic prism pair made of SF11 optical glass of $n'$ = 1.779 refractive index in air at 633 nm. At which angle of incidence ($\theta$) does the prism increase the beam width by a factor of two? How much should the refractive angle of the prism ($\varphi$) be, if we want the light to exit the other side perpendicularly?




$$\cos\theta' = \frac{d'}{a} \ ; \ \cos\theta = \frac{d}{a} \tag{10}$$

$$\frac{d'}{\cos\theta'} = \frac{d}{\cos\theta} \ ; \ d' = 2d \tag{11}$$

$$2\cos\theta = \cos\theta' \ \rightarrow \ 4\cos^2\theta = \cos^2\theta' \tag{12}$$

$$\sin\theta = n'\sin\theta' \ \rightarrow \ 4(1-\sin^2\theta) = 1 - \frac{\sin^2\theta}{n'^2} \tag{13}$$

$$3 = \left(4 - \frac{1}{n'^2}\right)\sin^2\theta \tag{14}$$

$$\sin\theta = \sqrt{\frac{3}{4 - \frac{1}{1.779^2}}} = 0.902 \ \rightarrow \ \theta = 64.5° \tag{15}$$

For a beam exiting perpendicularly:

$$\theta' = \varphi \tag{16}$$

$$\sin\theta' = \frac{\sin\theta}{n'} \ \rightarrow \ \theta' = 30.5° \tag{17}$$

## 6. Explanation of Brewster's effect by the lecture notes ...

## 7. Example for Brewster's effect: reflection on a glass sheet

Let us have a light beam (plane wave) of 550 nm incident from air on BK7 glass ($n'$ = 1.519). How much is Brewster's angle ($\theta_\mathrm{B}$) at the incident side? What is the surface reflectance ($R_s$)?

First we determine the formula necessary to calculate $\theta_\mathrm{B}$. According to the Fresnel formulae there is no p-polarization component at an incidence of Brewster's angle:

$$\rho_p = \frac{n' \cos\theta_{\mathrm{B}} - n\cos\theta'}{n'\cos\theta_{\mathrm{B}} + n\cos\theta'} = 0 \quad \Longrightarrow \quad n'\cos\theta_{\mathrm{B}} - n\cos\theta' = 0 \tag{18}$$

$$n'\cos\theta_{\mathrm{B}} = n\cos\theta' \tag{19}$$

From the equation above we still need to cancel the angle of refraction. For this reason we rearrange and square it:

$$\cos^2\theta' = \frac{n'^2}{n^2}\cos^2\theta_{\mathrm{B}} \tag{20}$$

The second equation needed to cancel $\theta'$ is the law of refraction by Snell:

$$n\sin\theta_{\mathrm{B}} = n'\sin\theta' \tag{21}$$

We have this rearranged and squared too:

$$\sin^2\theta' = \frac{n^2}{n'^2}\sin^2\theta_{\mathrm{B}} \tag{22}$$

Finally we add up (20) and (22):

$$\frac{n^2}{n'^2}\sin^2\theta_{\mathrm{B}} + \frac{n'^2}{n^2}\cos^2\theta_{\mathrm{B}} = \sin^2\theta' + \cos^2\theta' = 1 \tag{23}$$

The 1 value on the right side can be expressed by the sine and cosine of the angle of incidence too:

$$\frac{n^2}{n'^2}\sin^2\theta_{\mathrm{B}} + \frac{n'^2}{n^2}\cos^2\theta_{\mathrm{B}} = \sin^2\theta_{\mathrm{B}} + \cos^2\theta_{\mathrm{B}} \tag{24}$$

$$\left(\frac{n'^2}{n^2} - 1\right)\cos^2\theta_{\mathrm{B}} = \left(1 - \frac{n^2}{n'^2}\right)\sin^2\theta_{\mathrm{B}} \tag{25}$$

$$\left(\frac{n'^2 - n^2}{n^2}\right)\cos^2\theta_{\mathrm{B}} = \left(\frac{n'^2 - n^2}{n'^2}\right)\sin^2\theta_{\mathrm{B}} \tag{26}$$

From this the result:

$$\mathrm{tg}\,\theta_{\mathrm{B}} = \frac{n'}{n}. \tag{27}$$

We would arrive at exactly the same relationship if we examined the condition of having the refracted and reflected beams perpendicular to each other, which is thus equivalent with $\rho_{\mathrm{p}} = 0$. In this case then

$$\theta' + \theta_{\mathrm{B}} = 90° \tag{28}$$

Besides, Snell's law also satisfies, thus:

$$n\sin\theta_{\mathrm{B}} = n'\sin\theta' \;\; \rightarrow \quad n\sin\theta_{\mathrm{B}} = n'\sin(90° - \theta_{\mathrm{B}}) \tag{29}$$

$$n\sin\theta_{\mathrm{B}} = n'(1\cdot\cos\theta_{\mathrm{B}} - 0\cdot\sin\theta_{\mathrm{B}}) \tag{30}$$

$$\tan\theta_{\mathrm{B}} = \frac{n'}{n}. \tag{31}$$

From this: $\theta_{\mathrm{B}} = 56.6°$. We still need the angle of refraction. It is $\theta' = 33.4°$ according to (28). From this:

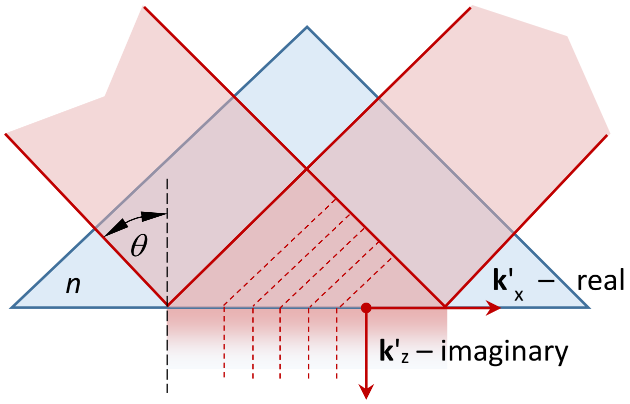
$$R_{\mathrm{s}} = |\rho_{\mathrm{s}}|^2 = \left|\frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'}\right|^2 = \left|\frac{0.550 - 1.519\cdot 0.835}{0.550 + 1.519\cdot 0.835}\right|^2 \tag{32}$$

From this: $R_{\mathrm{s}}$ = 15.6%, which means that though the reflected beam is polarized indeed, the efficiency of the effect is not too high.

## 8. Explanation of total internal reflection by the lecture notes ...

## 9. Example for total internal reflection: glass-air boundary surface

A plane wave of $\lambda_0$ = 550 nm propagates in BK7 glass ($n$ = 1.519), then arrives at an air interface ($n'$ = 1.0). What is the critical angle of TIR (total internal reflection) ($\theta_c$)?




At the critical angle $\theta'$= 90°, thus

$$n \sin\theta = n' \sin 90° \;\rightarrow\; \theta_c = \arcsin\frac{n'}{n} \tag{33}$$

Hence: $\theta_c$ = 41.2°. Let this plane wave strike the boundary surface at an angle of $\theta$= 45°. What is the phase shift of the reflected beam relative to that of the incident beam for s- and p-polarization?

$$\rho_s = \frac{1+i\gamma}{1-i\gamma} = e^{i\varphi_s} \text{ és } \rho_p = \frac{1+i\gamma\left(\frac{n}{n'}\right)^2}{1-i\gamma\left(\frac{n}{n'}\right)^2} = e^{i\varphi_p} \tag{34}$$

$$\gamma = \frac{\sqrt{n^2\sin^2\theta - n'^2}}{n\cos\theta} = \frac{\sqrt{1.519^2\cdot 0.5 - 1}}{1.519\cdot 0.707} = 0.365 \tag{35}$$

$$\varphi_s = 2\,\mathrm{arctg}\,\gamma = 40.1° \tag{36}$$

$$\varphi_p = 2\arctan\left(\gamma\left(\frac{n}{n'}\right)^2\right) = 2\arctan(0.365\cdot 1.519^2) = 80.2° \tag{37}$$

The difference of the two phases: $\varphi_p - \varphi_s$ = 40.1° = 0.223π [rad]

What is the penetration depth into air? Since

$$a \equiv \frac{k'_z}{k_z} = -i\gamma \;\rightarrow\; \tilde{k}'_z = 0 - ik_z\gamma \tag{38}$$

We defined the complex wave vector such as:

$$\tilde{k}'_z \equiv k'_{re} - ik'_{im} \;\rightarrow\; k'_{im} = k_z\gamma \tag{39}$$

$$k_z = \cos\theta\cdot n\frac{2\pi}{\lambda_0} \tag{40}$$

The penetration depth is:

$$\delta \equiv \frac{1}{k'_{im}} = \frac{1}{k_z\gamma} = \frac{\lambda_0}{\cos\theta\cdot n2\pi\gamma} = \frac{550}{0.707\cdot 1.519\cdot 2\pi\cdot 0.365} = 223 \text{ nm} \tag{41}$$